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B. TECH.
(SEM IV) THEORY EXAMINATION 2022-23
MATHEMATICS III

Time: 3 Hours ***Total Marks: 100***

Note: Attempt all Sections. If require any missing data; then choose suitably.

SECTION A

1. Attempt *all* questions in brief. 2 x 10 = 20

(a) Evaluate $\mathbf{L}[u_4(t)\ \sin nt\,]$, where $u_a(t)$ represents the unit step function.

(b) Describe the sufficient condition for the existence of Laplace transform of any function.

(c) If $U(z) = \frac{2z^2+5z+14}{(z-1)^4}$, evaluate u_2.Here $U(z)$represents the Z-transform of u_n .

(d) Write the complex form of Fourier integral representation of a function.

(e) Test whether the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 4 & 3 & 5 & 4 & 6 \end{pmatrix}$ is even permutation or odd permutation?

(f) State the Lagrange's theorem with reference to Group theory.

(g) In a class of 25 students, 12havetakenMathematics, 8 have taken Mathematics but not Biology. Find the number of students who have taken Mathematics & Biology both and also the number of students who have taken Biology but not Mathematics.

(h) If function $f: R \to R$ is defined as$f(x) = x^2$, find $f^{-1}(4)$ and $f^{-1}(-9)$.

(i) Define Lattice and Distributive lattice with examples.

(j) Prove the following absorption laws in Boolean algebra:

$(i)\ a + (a.b) = a (ii) a.(a + b) = a$

SECTION B

2. Attempt any *three* of the following: 10x3=30

(a) Apply the Convolution theorem to find the inverse Laplace transform $L^{-1}[\frac{s^2}{(s^2+a^2)(s^2+b^2)}]$

(b) Apply the Fourier transform to solve the Heat equation $\frac{\partial u}{\partial t} = 2\frac{\partial^2 u}{\partial x^2}$,

if $u(0,t) = 0,\ u(x,0) = e^{-x}, x > 0$ *and* $u(x,t)$ is bounded for $x > 0, t > 0$.

(c) Check the validity of the following argument:

If 2 is an odd number then 4 does not divide 10. Either 11 is not a prime number or 4 divides 10.

But 11 is a prime number, therefore 12 is an even number.

(d) Solve the following Recurrence relation:

$a_n - 7a_{n-1} + 10a_{n-2} = n + 2$ given $a_0 = 1, a_1 = 1$

(e) Convert the Boolean expression $[(xy' + xz)' + x']$ into Disjunctive normal form and Conjunctive normal form.

SECTION C

3. Attempt any *one* part of the following: 10x1=10

(a) Solve the following differential equation using Laplace transform:

$[D^3 - 3D^2 + 3D - 1]y = t^2 e^t$, given that $y(0) = 1, y'(0) = 0,\ y''(0) = -2$.

(b) Find the Laplace transform of the following function:

$$f(t) = \begin{cases} 0, & 0 \le t \le 1 \\ t-1, & 1 \le t \le 2 \\ 3-t, & 2 \le t \le 3 \\ 0, & t > 3 \end{cases}$$

4. Attempt any *one* part of the following: 10x1=10

(a) Find the Fourier transform of the following function:

$$f(x) = \begin{cases} 1-x^2, & |x| \le 1 \\ 0, & |x| > 1 \end{cases}$$ and hence find $\int_0^{\infty} \frac{x\cos x - \sin x}{x^3} \cos\frac{x}{2}\, dx$

(b) Solve the following difference equation using Z-transform:

$u_{n+2} - 5u_{n+1} + 6u_n = 2^n$ with $u_0 = 0, u_1 = 1$.

5. Attempt any *one* part of the following: 10x1=10

(a) Prove that the set of all positive rational numbers Q^+ forms an abelian group under the operation '$*$', defined as $a * b = \frac{ab}{2}$.

(b) Test whether the statement $p \Rightarrow (\sim q \vee r)$ is a tautology or a contradiction or a contingency.

Also, test whether $p \Rightarrow (\sim q \vee r) \equiv (p \Rightarrow \sim q) \vee (p \Rightarrow r)$.

6. Attempt any *one* part of the following: 10x1=10

(a) Show that $n^2 > (2n+1)$ for $n \ge 3$ by Mathematical Induction.

(b) Let R be a relation in a set of integers Z, defined by

$R = \{(x,y): x - y$ is divisible by $6,\ x \in Z, y \in Z\}$. Justify that R is an Equivalence relation or Partially ordered relation in Z.

7. Attempt any *one* part of the following: 10x1=10

(a) Let $A = \{1,2,3,4,6,8,9,12,18,24\}$ be any set. The relation '$\preccurlyeq$' in set A is defined as $x \preccurlyeq y$ when x divides y.

Is the set $(A, \preccurlyeq)$ partially ordered set (poset)? If yes, draw the Hasse diagram for this poset and hence list all the maximal, minimal, greatest and least elements of the poset.

(b) Draw a switching circuit for the following Boolean expression:

$[a + (b + c).a'].b' + c.[b.(a + c') + a.(b' + c)]$

Also draw a logic gate circuit for the minimized form of above Boolean expression.