B.TECH.

THEORY EXAMINATION (SEM–IV) 2016-17

MATHEMATICS-II

Time : 3 Hours ***Max. Marks : 70***

Note : *Be precise in your answer. In case of numerical problem assume data wherever not provided.*

SECTION – A

1. Attempt any seven parts for the following: 7 x 2 = 14

(a) Solve the differential equation $\frac{d^2y}{dx^2} = -12x^2 + 24x - 20$ with the condition x = 0, y = 5 and x = 0, y = 21 ad hence find the value of y at x = 1.

(b) For a differential equation $\frac{d^2y}{dx^2} + 2\alpha\frac{dy}{dx} + y = 0$, find the value of α for which the differential equation characteristic equation has equal number.

(c) For a Legend polynomial prove that P_n (1) = 1 and P_n (-1) = $(-1)^n$

(d) For the Bessel's function Jn(x) prove the following identities:
$J_{-n}(x) = (-1)^n J_n(x)$ and $J_{-n}(-x) = (-1)^n J_n(x)$

(e) Evaluate the Laplace transform of Integral of a function $L\left\{\int_0^t f(t/dt)\right\}$.

(f) Evaluate the value of integral $\int_0^\infty t.e^{-2t} cost\ dt.$

(g) Find the Fourier coefficient for the function $f(x) = x^2$ 0<x<2π

(h) Find the partial differential equation of all sphere whose centre lie on Z-axis.

(i) Formulate the PDE by eliminating the arbitrary function from $\phi(x^2 + y^2, y^2 + z^2) = 0$

(j) Specify with suitable example the clarification Partial Differential Equation (PDE) for elliptic, parabolic and hyperbolic differential equation.

SECTION – B

2. Attempt any three parts of the following questions: 3 x 7 = 21

(a) A function n(x) satisfies the differential equation $\frac{d^2n(x)}{dx^2} - \frac{n(x)}{L^2} = 0$, where L is a constant. The boundary conditions are n(0) = x and n(∞) = 0. Find the solution to this equation.

(b) Find the series solution by Forbenias method for the differential equation
$(1 - x^2)y" - 2xy' + 20y = 0$

(c) Determine the response of damped mass – spring system under a square wave given by the differential equation
$y" + 3y' + 2y = u(t-1) - u(t-2), \quad y(0) = 0, \quad y'(0) = 0$
Using the Laplace transform.

(d) Obtain the Fourier expansion of $f(x) = x\ sin\ x$ as cosine series in (0, π) and hence show that

$$\frac{1}{1\times 3} - \frac{1}{3\times 5} + \frac{1}{5\times 7} - \ldots\ldots\ldots\ldots. = \left(\frac{\pi - 2}{4}\right)$$

(e) Solve by method of separation of variable for PDE
$x\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0, \ \ u(x, 0) = 4e^{-x}$

SECTION – C

Attempt all parts of the following questions: **7 x 5 = 35**

3. Attempt any two parts of the following:

(a) Find the particular solution of the differential equation

$\frac{d^2y}{dx^2} + a^1 = \sec ax$

(b) If $y = y_1(x)$ and $y = y_2(x)$ are two solutions of the equation $\frac{d^2y}{dx^2} + P(x)\frac{dy}{dx} + Q(x)y = 0$, then show that $y_1\left(\frac{dy_2}{dx}\right) - y_2\left(\frac{dy_1}{dx}\right) = ce^{-\int Pdx}$, where c is constant.

(c) Solve by method of variation of Parameter for the differential equation :

$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + ay = \left(\frac{e^{3x}}{x^2}\right)$

4. **Attempt any two parts of the following:**

(a) Prove that $\sqrt{\frac{\pi x}{2}}.J_{3/2}(x) = \left(\frac{1}{x}\sin x - \cos x\right)$

(b) Show that Legendre polynomials are orthogonal on the interval [-1, 1]

(c) Prove that $\int_{-1}^{+1} xP_n(x)dx = \frac{2n}{4n^2-1}$

5. **Attempt any two parts of the following:**

(a) Find the Laplace transform of S_{RW} – tooth wave function

F(t) = Kt in 0 < t < 1 with period 1

(b) Use Convolution theorem to find the inverse of function $F(s) = \frac{4}{s^2+2s+5}$

(c) Solve the simultaneous differential equation, using Laplace transformation –

$\frac{dy}{dt} + 2x = \sin 2t;\ \frac{dy}{dt} - 2y = \cos 2t$, where x (0) = 1, y (0) = 0

6. **Attempt any two parts of the following:**

(a) If $f(x) = \left[\frac{\pi - x}{2}\right]^2$, 0 < x < 2π then show that $f(x) = \frac{\pi^2}{12} - \sum_{n=1}^{\infty}\frac{1}{n^2}\cos nx$

(b) Find the complete solution of PDE

$(\Delta^2 + 7\Delta D' + 12D'^2)/2 = \sin hx$, where symbols have their usual meaning.

(c) Solve the PDE $p + 3q = 5z + \tan(y - 3x)$

7. **Attempt any one part of the following:**

(a) A square plate is bounded by lines x = 0, y = 0; x = 20, y = 20. Its faces are insulated. The temperature along the upper horizontal edge is given by u (x, 20) = x (20 – x) when 0 < x < 20 while the upper three edges are kept at 0^0C. Find the steady state temperature.

(b) A bar of 10 cm long with insulated sides A and B are kept at 20^0C and 40^0C respectively until steady state conditions prevail. The temperature at A is then suddenly varies to 50^0C and the same instant that at B bowered to 10^0C. Find the subsequent temperature at any point of the bar at any time.